Eutocius's Commentary on Cube Duplication

In his commentary on Archimedes's *Sphere and Cylinder II,* Eutocius provides a number of solutions given by various authors to the problem of finding two mean proportionals. Specifically, he writes:

"We do not find written by him [sc. Archimedes] anything at all on the finding of these things, but we have come upon writings of many ingenious men, which report on this problem… In order that the conception of those men who are accessible to us becomes clear, the manner of the discovery of each will also here be written down" (Knorr, 1989: 77).

Eutocius then offers a catalogue of solutions, the second of which is attributed to Plato.

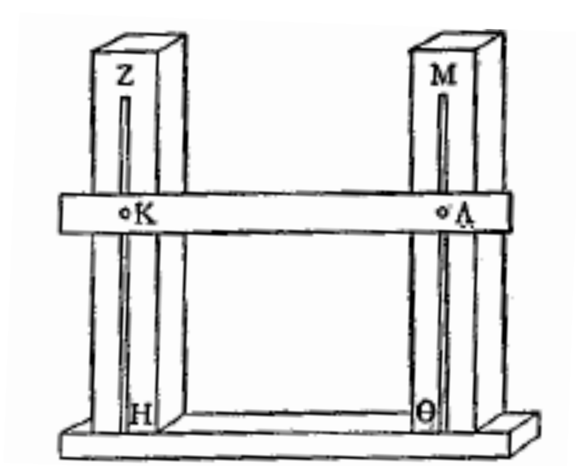
<u>As Plato</u>

The solution (re-written for clarity):

1. Draw the two given lines whose two mean proportionals we are searching for at right angles to one another. Let the angle be *ABΓ*.
2. Let the line section *AB* continue toward *Δ*, and the line section *ΓB* continue toward *E*.

Leave the diagram behind and begin constructing an instrument (or at least pretend to):

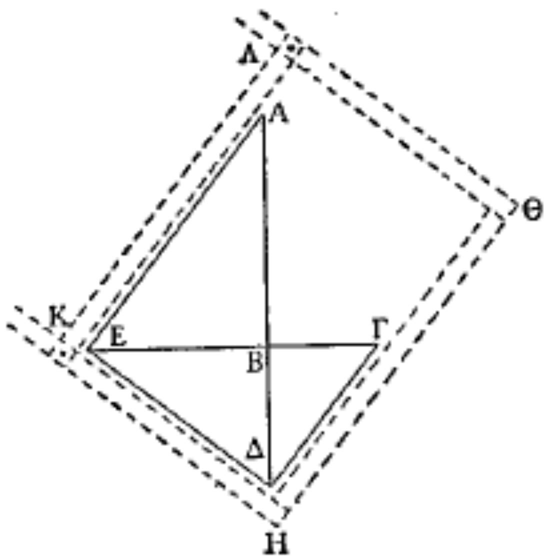
3. Take two rods of wood and construct a right angle *ZHΘ*.
4. Take an additional rod of wood and place it at *Θ* parallel to *ZH*, call it *ΘM*.
5. Create grooves in both *ZH* and *ΘM*.
6. Take a ruler *KΛ,* whose length is equal to *HΘ*, and place a knob in each of its ends, so it will fit within the grooves just created. The ruler *KΛ* therefore can move up and down while remaining perpendicular to *HΘ*.



7. Now take the instrument and place it (on the diagram) so that the side *HΘ* touches the point *Γ* .

8. Once the instrument is so placed, rotate it around and move the ruler *KΛ* up and down until the following conditions are met. The point *H* is directly on the line *BΔ* (this side *HΘ* must still touch the point *Γ*), and the ruler *KΛ* is placed in such a way that the point *K* in on the line *BE*, and it touches the point *A*.

This gives a complete solution to the problem as the two lines created, *BΔ* and *BE,* are the two desired proportionals, so that *ΓB:BΔ=BΔ:BE=BE:AB.*



Commentary:

According to both Knorr (1989) and Netz (2003), the attribution of this solution to Plato must be a mistake. Knorr notes that Plato would have objected to the mechanical nature of the solution, not invented it. And Netz points out that it is unlikely that a mathematical text by Plato, which survived from antiquity until the sixth century AD, left no other trace. Netz also claims that it is highly unlikely that Eutocius referred to any other ancient author by the same name, or as he put it, "a Plato is a Plato is a Plato."

"Plato's solution" is the second one Eutocius mentions. The first was a false solution he attributes to Eudoxus of Cnidus. Specifically, Eutocius writes that while in the preface to his solution Eudoxus mentions curved lines, the proof itself not only does not use curved lines, but also finds a discrete rather than a continuous proportion. He writes, "this would be absurd to imagine, not to mention for Eudoxus, but for anyone even moderately versed in geometry" (Knorr, 1989: 78).

Knorr proposes that "Plato's solution" in fact belongs to Eudoxus. He suggests that the method was transmitted by Eratosthenes, and that "he might have introduced it as a centerpiece of a discussion on the nature of geometry" (Knorr, 1986: 59) between Plato and Eudoxus. As Eutocius writes, Eudoxus's method involved curves, and with a slight alteration of the procedure detailed above, the mechanical instrument can indeed create curved lines. Thus, he proposes that Eudoxus had a theoretical discussion of the problem, in which “Plato’s construction” is a specific case.

Netz (2008) offers a different interpretation of the circumstances. He argues that it is highly unlikely that Eudoxus was the author of the concrete solution known as Plato’s construction. Eudoxus’s solution was said to be highly abstract, and there is no evidence leading one to believe that some other mathematician came up with this application of Eudoxus and attributed it to him instead of taking the credit for himself. Moreover, Netz claims that Knorr’s explanation does not account for what might be the source of the false solution that Eutocius attributes to Eudoxus. Finally, Netz writes that false solutions are quite rare in ancient sources, and “is likely to be transmitted, copied, and re-copied only if it has some external, non-mathematical interest” (504).

Instead, Netz goes back to Plato’s construction of the Divided Line in the *Republic:*

“Now take a line which has been cut into two unequal parts, and divide each of them again in the same proportion, and suppose the two main divisions to answer, one to the visible and the other to the intelligible, and then compare the subdivisions in respect of their clearness and want of clearness.”

Netz points out that in a certain sense this construction can be seen as a false solution to the problem of finding two mean proportionals. Namely, following the construction you end up with the following: $(A:B)::(C:D)::((A+B):(C+D))$. This is obviously a false solution, but it is one that is associated with Plato and which held “non-mathematical interest.”

Netz suggests therefore that “a late ancient author could have compiled a list of several solutions to the problem of finding two mean proportionals, putting in a mathematized version of the Divided Line passage, describing this as ‘Plato’s solution’” (507).

Netz goes on to reconstruct a possible explanation. Assuming that the original form of Eutocius’s source contained the following information (reproduced from Netz):

‘As Eudoxus’	Introduction by Eudoxus	Solution by Eudoxus
‘As Plato’	Solution by Plato (= mathematized Divided Line)	
‘As X’	Solution by X (= ‘Plato’s solution’)	

Then, if in the source Eutocius was consulting, the solution by Eudoxus for some reason disappeared (“a major oversight, but not at all an impossibility”), the structure will become:

‘As Eudoxus’	Introduction by Eudoxus
	Solution by Plato (= mathematized Divided Line)
‘As Plato’	Solution by X (= ‘Plato’s solution’).

Netz concludes:

"Finally, an apology for my own article. The argument offered here is tentative, in the sense that one cannot tell how much the story offered here is likely to be true. But a truth has been shown, I believe, through the chain of argument itself. We have followed possibilities and plausibilities: they need not be true, but the fact of their being plausible or possible is a reality well worth learning. We have followed the shadows of ancient events to whose reality we may never have access, learning, in the process, something real about Platonic mathematical philosophy and about its ancient transmissions and transformations" (509).

Sources:
Netz, Raviel. 2003. "Plato's Mathematical Construction." *The Classical Quarterly* 53(2).

Knorr, Wilbur Richard. 1986. *The Ancient Tradition of Geometric Problems.*

Knorr, Wilbur Richard. 1989. "Eutocius's Anthology of Cube Duplication," in *Textual Studies in Ancient and Medieval Geometry.*